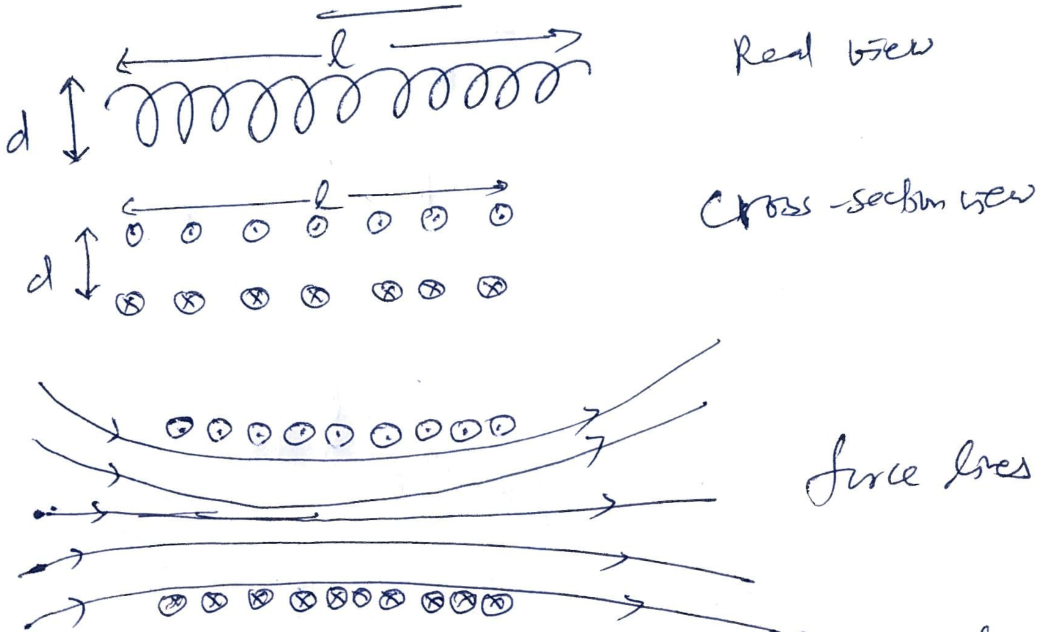
Inductance of a Solenoid (Infinite length)

Solenoid → long copper wire in the form of helix such that its length (l) is large in comparison to its diameter (d)

$l >> d$



The windings of solenoid are close packed. In each winding magnetic ~~flux~~ field is produced.

The magnetic field on the axis of coil due to flow of current i is given by, air core between the windings.

$$B = \mu_0 \frac{Ni}{l}$$

$N \rightarrow$ windings (turns)
$l \rightarrow$ length

If $A \rightarrow$ area of cross section
magnetic flux in each turn

$$\phi_l = BA$$

$$\phi_l = \mu_0 \frac{NiA}{l}$$

magnetic flux in complete solenoid

$$\phi_N = N \cdot \phi_l$$

$$= \mu_0 \frac{N^2}{l} A i$$

If the current 'i' changes, the flux also changes → the resultant e.m.f is

$$\varepsilon_{ind} = (-)\frac{d\phi_n}{dt}$$

$$= -\frac{d}{dt}\left[\mu_0 \frac{N^2}{l} A i\right]$$

$$= (-)\mu_0 \frac{N^2 A}{l} \cdot \frac{di}{dt} \quad \text{--- (1)}$$

from definition

$$\varepsilon_{ind} = (-) L \frac{di}{dt} \quad \text{--- (2)}$$

$$\Rightarrow \quad L = \frac{\mu_0 N^2 A}{l}$$

$$\boxed{L = \mu_0 n^2 l A}$$

here $n = \frac{N}{l}$ → No. of turns in unit length

If there is iron core or any other medium in the solenoid

$$L = \mu n^2 l A$$

where $\mu = \mu_0 \mu_r$ $\quad \mu_r$ → relative permeability of core

The energy stored in inductor is in magnetic field. The energy stored in self inductance

$$W = \frac{1}{2} L I^2$$

$$\Rightarrow \quad W = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right) I^2$$

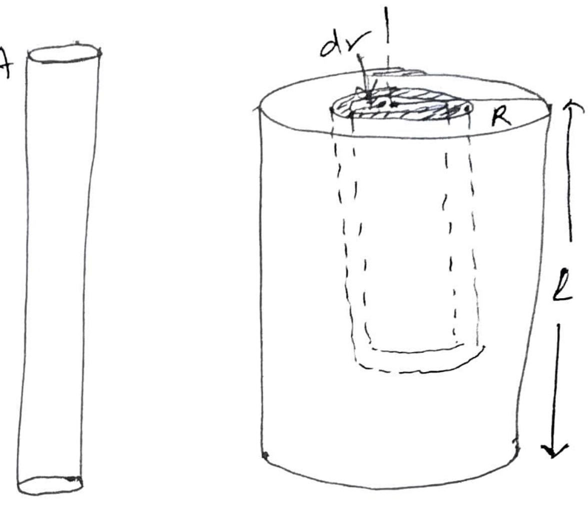
The energy in unit volume (lA)

Energy density $U = \frac{W}{lA} = \frac{1}{2}\mu_0\left(\frac{N^2 I^2}{l^2}\right)$

$$U = \frac{B^2}{2\mu_0} \quad \because \frac{N^2 I^2}{l^2} = \frac{B^2}{\mu_0^2}$$

Inductance of a long Straight Conductor

i current passing through a long straight conductor AB. We show magnified section of the wire also.



Let $R \rightarrow$ radius

and $l \rightarrow$ length of wire

Let we imagine a cylinder of thickness dr at r distance from the axis.

The differential magnetic flux due to current i in the conductor

$$d\phi_m = \frac{\pi r^2}{\pi R^2}[B(r)]\, l\, dr \quad \text{---} (1)$$

The magnetic field at r distance from the axis of conductor of radius R

$$B(r) = \frac{\mu_0 I r}{2\pi R^2}$$

$$\Rightarrow d\phi_m = \frac{r^2}{R^2}\left[\frac{\mu_0 I r}{2\pi R^2}\right] l\, dr$$

$$= \frac{\mu_0 I l}{2\pi R^4} r^3 dr \quad \text{---} (2)$$

$\Rightarrow$ The Total differential magnetic flux

$$\phi_m = \int d\phi_m = \frac{\mu_0 I l}{2\pi R^4}\int_0^R r^3 dr$$

$$\Rightarrow \phi_m = \frac{\mu_0 I l}{2\pi R^4}\left[\frac{r^4}{4}\right]_0^R = \frac{\mu_0 I l}{8\pi} \quad \text{---} (3)$$

If $L \rightarrow$ self inductance of wire

$$L = \frac{\phi_m}{I}$$

$$\Rightarrow L = \frac{\mu_0 I l}{8\pi I} = \frac{\mu_0 l}{8\pi} \quad \text{---(4)}$$

The self inductance per unit length

$$\boxed{L' = \frac{L}{l} = \frac{\mu_0}{8\pi}} \quad \text{---(5)}$$

Energy Stored in Magnetic field of a current carrying Solenoid

Energy stored in a current carrying coil

$$U_B = \frac{1}{2} L I_0^2 \quad \text{---(1)}$$

Self inductance of a solenoid

$$L = \frac{\mu_0 N^2 A}{l}$$

Magnetic field inside a solenoid

$$B = \frac{\mu_0 N i_0}{l} \quad \text{or} \quad i_0 = \frac{B l}{\mu_0 N}$$

From eqn (1) $$U_B = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \frac{B^2 l^2}{\mu_0^2 N^2}$$

$$\therefore U_B = \frac{1}{2} \frac{B^2 l A}{\mu_0} \text{ Joule} \quad \text{---(2)}$$

$A l \rightarrow$ Volume of solenoid

Energy stored per unit volume

$$\text{Energy density} = \frac{U_B}{Al} = \frac{1}{2} \frac{B^2}{\mu_0} \text{ Joules}/m^3 \quad \text{---(3)}$$

Applicable for all current carrying ~~conductor~~ inductors

The energy stored in electric field is

$$\frac{1}{2} \varepsilon_0 E^2$$